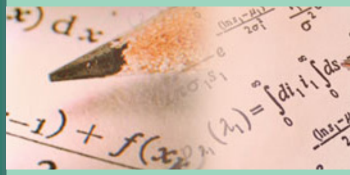
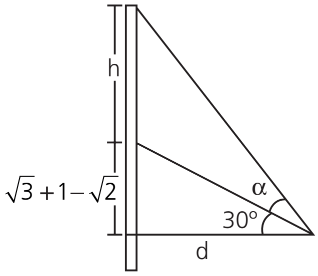


01. Temos que:



I. $\alpha = 45 \cdot 0{,}5° = 22{,}5°$

II. $tg(22{,}5° + 22{,}5°) = \dfrac{tg\alpha + tg\alpha}{1 - tg\alpha \cdot tg\alpha} \to 1 = \dfrac{2tg\alpha}{1 - tg^2\alpha} \to 2tg\alpha = 1 - tg^2\alpha \to tg^2\alpha + 2tg\alpha - 1 = 0 \to tg\alpha = \dfrac{-2 \pm \sqrt{8}}{2} \to \boxed{tg\alpha = -1 + \sqrt{2}}$

III. $tg = 30° = \dfrac{\sqrt{3}+1-\sqrt{2}}{d} \Rightarrow \dfrac{\sqrt{3}}{3} = \dfrac{\sqrt{3}+1-\sqrt{2}}{d} \Rightarrow d = \dfrac{(\sqrt{3}+1-\sqrt{2}) \cdot 3}{\sqrt{3}} = (\sqrt{3}+1-\sqrt{2})\sqrt{3}$

IV. $tg(30° + \alpha) = \dfrac{tg\,30° + tg\alpha}{1 - tg30° \cdot tg\alpha} \to \dfrac{h + \sqrt{3} + 1 - \sqrt{2}}{d} = \dfrac{\frac{\sqrt{3}}{3} + (-1+\sqrt{2})}{1 - \frac{\sqrt{3}}{3} \cdot (-1+\sqrt{2})} \to \dfrac{h + \sqrt{3} + 1 - \sqrt{2}}{d} = \dfrac{\frac{\sqrt{3} - 3 + 3\sqrt{2}}{3}}{\frac{3 + \sqrt{3} - \sqrt{6}}{3}} \to \dfrac{h + \sqrt{3} + 1 - \sqrt{2}}{(\sqrt{3}+1-\sqrt{2})\sqrt{3}}$

$= \dfrac{\sqrt{3} - 3 + 3\sqrt{2}}{3 + \sqrt{3} - 6} \to h + \sqrt{3} + 1 - \sqrt{2} = \sqrt{3} - 3 + 3\sqrt{2} \to h = 4\sqrt{2} - 4 \to h = 4(\sqrt{2} - 1)$

Resposta: C

02. Sendo S a soma das áreas das figuras, temos:

$S = \dfrac{\cos a \cdot \operatorname{sen} b}{2} + \dfrac{\cos a \cdot \operatorname{sen} b}{2} + \operatorname{sen} a \cdot \cos b + \operatorname{sen} a \cdot \cos b + \dfrac{\cos a \cdot \operatorname{sen} b}{2} + \dfrac{\cos a \cdot \operatorname{sen} b}{2}$

$S = 4 \cdot \left(\dfrac{\cos a \cdot \operatorname{sen} b}{2}\right) + 2\,\operatorname{sen} a \cdot \cos b$

$S = 2 \cdot [\operatorname{sen} a \cdot \cos b + \operatorname{sen} b \cdot \cos a]$

$S = 2 \cdot \operatorname{sen}(a + b)$

$S = 2 \cdot \operatorname{sen}\dfrac{\pi}{6} = 2 \cdot \dfrac{1}{2} = 1$

Resposta: D

03. Temos:

$Exp = \dfrac{1}{\cos 80°} - \dfrac{\sqrt{3}}{\operatorname{sen} 80°} \Rightarrow Exp = \dfrac{1}{\cos 80°} - \dfrac{tg60°}{\operatorname{sen} 80°} \Rightarrow Exp = \dfrac{1}{\cos 80°} - \dfrac{\operatorname{sen} 60°}{\cos 60° \operatorname{sen} 80°} \Rightarrow Exp = \dfrac{\operatorname{sen} 80° \cos 60° - \operatorname{sen} 60° \cos 80°}{\cos 60° \operatorname{sen} 80° \cos 80°} \Rightarrow$

$Exp = \dfrac{2\operatorname{sen} 20°}{\operatorname{sen} 80° \cos 80°} = \dfrac{4\operatorname{sen} 20°}{\operatorname{sen} 160°} = 4$

Resposta: D

04.

- α e β pertencem ao 1º quadrante $\to$ Razões trigonométricas positivas
- $tg\alpha = \dfrac{3}{4} \to \operatorname{sen}\alpha = \dfrac{3}{4}$ e $\cos\alpha = \dfrac{4}{5}$
- $\sec\beta = \dfrac{13}{5} \to \cos\beta = \dfrac{5}{13}$ e $\operatorname{sen}\beta = \dfrac{12}{13}$

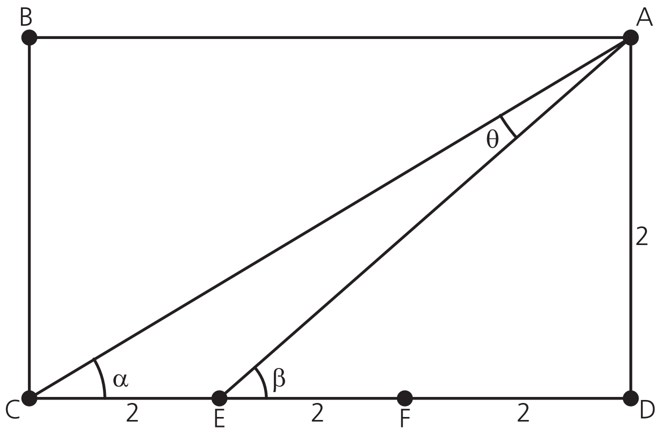
Assim:
$\operatorname{sen}(\alpha + \beta) = \operatorname{sen}\alpha \cos\beta + \operatorname{sen}\beta \cos\alpha$

$\operatorname{sen}(\alpha + \beta) = \dfrac{3}{5} \cdot \dfrac{5}{13} + \dfrac{12}{13} \cdot \dfrac{4}{5} = \dfrac{63}{65}$

Logo: $65\operatorname{sen}(\alpha + \beta) = 63$

Resposta: D

05.



- $\beta = \alpha + \theta$ (Teorema de ângulo externo)

Sabemos que:

$$\text{tg}(\alpha+\theta) = \frac{\text{tg}\alpha + \text{tg}\theta}{1-\text{tg}\alpha\cdot\text{tg}\theta} = \text{tg}\beta$$

Assim:

$$\frac{\frac{2}{6}+\text{tg}\theta}{1-\frac{2}{6}\cdot\text{tg}\theta} = \frac{2}{4} \rightarrow \text{tg}\theta = \frac{1}{7}$$

Resposta: E

06. Temos:

I) $x^2 - (\text{tg}\beta + \text{cotg}\beta)x + 1 = 0$

$x^2 + 1 = (\text{tg}\beta + \text{cotg}\beta)x$

$$\frac{x^2+1}{x} = \text{tg}\beta + \text{cotg}\beta$$

$$\frac{x^2+1}{x} = \frac{\text{sen}\beta}{\cos\beta} + \frac{\cos\beta}{\text{sen}\beta}$$

$$\frac{x^2+1}{x} = \frac{1}{\text{sen}\beta\cos\beta}$$

$$\text{sen}\beta\cos\beta = \frac{x}{x^2+1}$$

II) Como $2+\sqrt{3}$ é raiz, vem:

$$\text{sen}\beta\cos\beta = \frac{2+\sqrt{3}}{\left(2+\sqrt{3}\right)^2+1}$$

$$\text{sen}\beta\cos\beta = \frac{2+\sqrt{3}}{8+4\sqrt{3}} = \frac{1}{4}$$

Portanto:

$$\text{Exp.} = 392\ \text{sen}\beta\cos\beta = 392 \cdot \frac{1}{4} = 98$$

Resposta: E

07. Temos:

$$\text{tg}x \cdot \text{tg}\left(\frac{\pi}{3}-x\right)\cdot \text{tg}\left(\frac{\pi}{3}+x\right) = \text{tg}3x$$

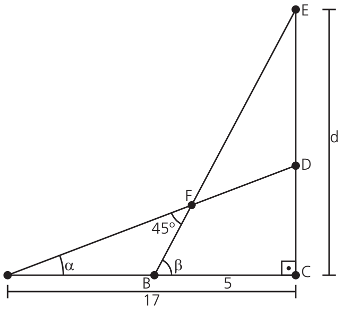
Fazendo $x = 20°$, encontramos:

$\text{tg}20°\cdot\text{tg}(60°–20°)\cdot\text{tg}(60°+20°) = \text{tg}60°$

$\text{tg}20°\cdot\text{tg}40°\cdot\text{tg}80° = \sqrt{3}$

Resposta: D

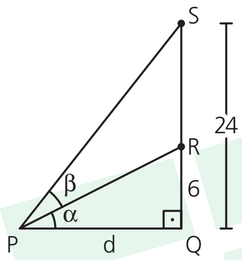
08. Temos:



- $\beta = \alpha + 45°$ (teorema do ângulo externo)
- $tg\beta = tg\,(\alpha + 45°) = \dfrac{tg\alpha + tg45°}{1 - tg\alpha \cdot tg45°} = \dfrac{\dfrac{7}{17}+1}{1-\dfrac{7}{17}\cdot 1} = \dfrac{12}{5}$
- $tg\beta = \dfrac{d}{5} = \dfrac{12}{5}$ $(\Delta BCE) \rightarrow d = 12$

Resposta: C

09. Temos:



I. $\alpha = \beta \Rightarrow tg(2\alpha) = \dfrac{24}{d}$

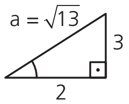
II. $tg\alpha = \dfrac{6}{d}$

III. $tg(\alpha+\alpha) = \dfrac{tg\alpha + tg\alpha}{1 - tg\alpha \cdot tg\alpha} \Rightarrow \dfrac{24}{d} = \dfrac{\dfrac{6}{d}+\dfrac{6}{d}}{1-\dfrac{36}{d^2}} \Rightarrow 24\cdot\left(1-\dfrac{36}{d^2}\right) = d\cdot\left[\dfrac{6}{d}+\dfrac{6}{d}\right] \Rightarrow 24\left(\dfrac{d^2-36}{d^2}\right) = 6+6 \Rightarrow 24(d^2-36) = 12d^2 \Rightarrow 12d^2 = 24\cdot 36 \Rightarrow$

$\Rightarrow d^2 = 2\cdot 36 \Rightarrow d = 6\sqrt{2} \Rightarrow d \cong 6\cdot 2{,}41 = 8{,}46\text{ m} \cong 8{,}5\text{ m}$

Resposta: A

10. Considere o triângulo retângulo de catetos 3 e 2 seguinte.



- $a^2 = 2^2 + 3^2 \rightarrow a = \sqrt{13}$

Nesse caso, temos que:

$f(x) = 3\cos x + 2\,\text{sen}\,x$

$f(x) = \sqrt{13}\cdot\left(\dfrac{3}{\sqrt{13}}\cos x + \dfrac{2}{\sqrt{13}}\text{sen}\,x\right)$

Tomemos:

$\text{sen}\,\alpha = \dfrac{3}{\sqrt{13}}$ e $\cos\alpha = \dfrac{2}{\sqrt{13}}, \alpha \in \left(0, \dfrac{\pi}{2}\right)$

Então:

$f(x) = \sqrt{13}\cdot(\text{sen}\,\alpha\cos x + \text{sen}\,x\cos\alpha)$

$f(x) = \sqrt{13}\cdot\text{sen}(\alpha + x)$

Logo:

$f_{máx.} = \sqrt{13}$

$f_{mín.} = -\sqrt{13}$

Resposta: C