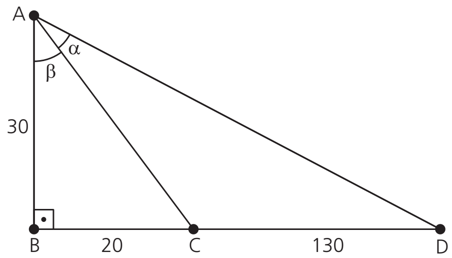
Matemática V

Aula 06: Transformações Trigonométricas Adição e Subtração de Arcos

Exercícios de Fixação

01. Temos:



$tg\beta = \frac{20}{30} = \frac{2}{3}$

$tg(\alpha+\beta) = \frac{150}{30} = 5$

Usando a fórmula $tg(\alpha+\beta) = \frac{tg\alpha + tg\beta}{1 - tg\alpha \cdot tg\beta}$, concluímos: $5 = \frac{tg\alpha + \frac{2}{3}}{1 - tg\alpha \cdot \frac{2}{3}} \rightarrow tg\alpha = 1 \rightarrow \alpha = 45°$

Resposta: B

02. Como

$$\begin{aligned} sen15° &= sen(45° - 30°) \\ &= sen45°\ cos30° - sen30°\ cos45° \\ &= \frac{\sqrt{2}}{2} \cdot \frac{\sqrt{3}}{2} - \frac{1}{2} \cdot \frac{\sqrt{2}}{2} \\ &= \frac{\sqrt{6} - \sqrt{2}}{4} \end{aligned}$$

Então:

$sen15° = \frac{h_1}{a} \Leftrightarrow h_1 = \frac{a(\sqrt{6} - \sqrt{2})}{4}.$

Além disso,

$sen45° = \frac{h_2}{a} \Leftrightarrow h_2 = \frac{a\sqrt{2}}{2}$

Então:

$$\begin{aligned} h_1 + h_2 &= \frac{a(\sqrt{6} - \sqrt{2})}{4} + \frac{a\sqrt{2}}{2} \\ &= \frac{a(\sqrt{6} + \sqrt{2})}{4}. \end{aligned}$$

Por outro lado,

$$\begin{aligned} sen75° &= sen(45° + 30°) \\ &= sen45°\ cos30° + sen30°\ cos45° \\ &= \frac{\sqrt{2}}{2} \cdot \frac{\sqrt{3}}{2} + \frac{1}{2} \cdot \frac{\sqrt{2}}{2} \\ &= \frac{\sqrt{6} + \sqrt{2}}{4} \end{aligned}$$

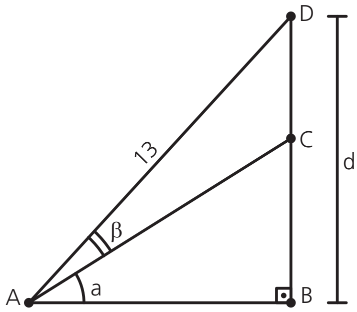
Então:

$sen75° = \frac{h_3}{a} \Leftrightarrow h_3 = \frac{a(\sqrt{6} + \sqrt{2})}{4}.$

Portanto, $h_1 + h_2 = h_3$

Resposta: D

03.



$\cos\alpha = \frac{4}{5} \to \text{sen}\alpha = \frac{3}{5}\ (\alpha \text{ agudo})$

$\cos\beta = \frac{12}{13} \to \text{sen}\beta = \frac{5}{13}(\beta \text{ agudo})$

$\text{sen}(\alpha+\beta) = \text{sen}\alpha\cos\beta + \text{sen}\beta\cos\alpha = \frac{3}{5}\cdot\frac{12}{13} + \frac{5}{13}\cdot\frac{4}{5} = \frac{56}{65}$

Logo: $\frac{d}{13} = \frac{56}{65} \to d = \frac{56}{5} = 11{,}2$

Resposta: A

04. De acordo com os dados do problema, temos o sistema:

$$\begin{cases} x - y = 60 \\ x + y = 90 \end{cases}$$

Resolvendo o sistema temos x = 75° e y = 15°. Assim, temos:

I. $\text{sen}75° = \text{sen}(45° + 30°)$

$= \frac{\sqrt{2}}{2}\cdot\frac{\sqrt{3}}{2} + \frac{1}{2}\cdot\frac{\sqrt{2}}{2} = \frac{\sqrt{6}+\sqrt{2}}{4}$

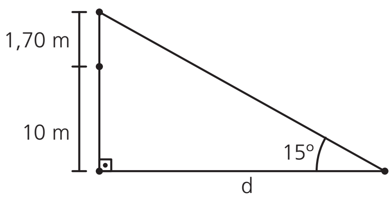
II. $\text{sen}15° = \text{sen}(45° - 30°)$

$= \frac{\sqrt{2}}{2}\cdot\frac{\sqrt{3}}{2} - \frac{1}{2}\cdot\frac{\sqrt{2}}{2} = \frac{\sqrt{6}-\sqrt{2}}{4}$

Daí, $\text{sen}75° + \text{sen}15° = \frac{2\sqrt{6}}{4} = \frac{\sqrt{6}}{2}$

Resposta: C

05. De acordo com o enunciado, temos:



$$\text{tg}15° = \frac{11{,}7}{d} = \frac{\text{tg}45° - \text{tg}30°}{1 + \text{tg}45°\cdot\text{tg}30°} = \frac{1 - \frac{\sqrt{3}}{3}}{1 + \frac{\sqrt{3}}{3}}$$

Simplificando:

$$\frac{11{,}7}{d} = \frac{3-\sqrt{3}}{3+\sqrt{3}} \to d = \frac{11{,}7\cdot(3+\sqrt{3})}{3-\sqrt{3}} \to d = \frac{11{,}7\cdot(3+\sqrt{3})^2}{6} \simeq 43\text{m}$$

Resposta: C